THEORY AND OBJECTIVES OF AIR DISPERSION MODELLING

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1. Introduction and Objectives

Air pollution models are routinely used in environmental impact assessments, risk analysis and emergency planning, and source apportionment studies. In highly polluted cities such as Athens, Los Angeles and Mexico, regional scale air quality models are used to forecast air pollution episodes – the results from these models may initiate compulsory shutdown of industries or vehicle restrictions. The various roles served by air pollution models, which cover a broad range of scales from local to global, lead to distinct modelling requirements. The focus of this review will be on the near-field impact (< 10-20 km) of industrial sources. The emphasis is on Gaussian-plume type models for continuous releases, which are at the core of most U.S. Environmental Protection Agency (EPA) regulatory models.

Nowadays the term "air pollution model" usually refers to a computer program, but in the past it has also included hand calculations or use of charts and tables from simple handbooks. A dispersion model is essentially a computational procedure for predicting concentrations downwind of a pollutant source, based on knowledge of the emissions characteristics (stack exit velocity, plume temperature, stack diameter, *etc*.), terrain (surface roughness, local topography, nearby buildings) and state of the atmosphere (wind speed, stability, mixing height, *etc*.). Figure 1 illustrates the flow of information in a generic air pollution model. The basic problem is to predict the rate of spread of the pollutant cloud, and the consequent decrease in mean concentration. The model has to be able to predict rates of diffusion based on measurable meteorological variables such as wind speed, atmospheric turbulence, and thermodynamic effects. The algorithms at the core of air pollution models are based upon mathematical equations describing these various phenomena which, when combined with empirical (field) data, can be used to predict concentration distributions downwind of a source.

The modern science of air pollution modelling began in the 1920's when military scientists in England tried to estimate the dispersion of toxic chemical agents released in the battlefield under various conditions. This early research is summarized in the groundbreaking textbook by Sutton (1953). Rapid developments in the 1950's and 1960's, including major field studies and advances in the understanding of the structure of the atmosphere, led to the development of the first regulatory air pollution models in the U.S. The textbooks by Pasquill (1974) and Stern (1976) review much of the research and theory up until the mid 1970's. However, the proliferation of air pollution research and models to date has made it necessary to read specialized journals and conference proceedings to keep up with developments. This is not practical for all model users, and so the present workshop has been designed to help bridge the gap between the basic concepts of dispersion theory and the sophisticated theories used in advanced USEPA models such as ISC3 and AERMOD-PRIME. This paper reviews the fundamentals of Gaussian plume modelling as normally presented in an undergraduate air pollution course. Many of the key concepts and algorithms incorporated into advanced air dispersion models are briefly explained.

2. Model Requirements and Model Selection

There are several competing requirements in the design of an air pollution model. A model must capture the essential physics of the dispersion process and provide reasonable and repeatable estimates of downwind concentrations. This generally requires detailed knowledge of source characteristics, terrain and meteorology, but it is also desirable to keep these input requirements to a minimum, and simplicity is an important asset in any model. All models should have a fully documented account of the equation algorithms used and their conversion into valid software (*i.e.*, traceability). Regulatory models must also undergo extensive quality assurance, including the evaluation of the model under several scenarios using benchmark data. Standard statistical procedures have been developed for expressing the uncertainty and variability of the predicted results when comparing them to measured concentrations (*e.g.*, Hanna, 1989).

In choosing an air dispersion model, several levels of model are available, with progressively increasing levels of mathematical sophistication, input data requirements and user expertise required. At the low end of the scale are the **gross screening models**, which require only a hand-held calculator, nomograph, or spreadsheet. They may treat only one source at a time (*e.g.*, a single elevated stack) and provide some sort of worst-case prediction based on relatively primitive meteorological information. It is often wise to apply such a model prior to using the more advanced models, where the flow of information is more difficult to follow.

Next on the scale of model complexity are **intermediate models**, usually PC-based, which may include varying meteorology (wind speed and stability classes) and more sophisticated source information. Many early EPA models fall in this category, including the SCREEN3 model.

Advanced models require a desktop PC or workstation. They require extensive data sets for meteorology and emissions, and include multiple source types - point, area and volume. They may also include additional features such as complex terrain, flow around buildings and layered atmospheric structure. Modern models incorporate the most up-to-date treatment of the atmosphere such as Monin-Obukhov similarity theory. Some examples of advanced models are the EPA models ISC3, AERMOD and CALPUFF, the British Model ADMS (Carruthers *et al*., 1994) and the Danish model OML (Berkowicz *et al*., 1987)

Specialized models are often used for predicting dispersion of special hazardous materials, such as military models used in chemical/biological defense. Heavy gas dispersion models are used by the chemical process industries to model the behaviour of rogue or accidental releases of dense gases or vapours. These models may require extensive thermodynamic information to account for release conditions. Models such as SLAB and DEGADIS (Dense Gas Dispersion Model) models are typical of this family.

Although the input data requirements and level of sophistication increase with the more advanced models, a more complex model does not necessarily lead to predictions that are more accurate. As the number of input variables goes up in the advanced models, the room for input data error increases. In addition, the level of user understanding must increase to make proper use of the model.

3. Example Gross Screening Analysis

It is often useful to perform a simple screening analysis *before* applying a more refined computer analysis. A gross screening analysis will quickly identify the order of magnitude of the expected concentrations and may even show that no problem exists, in which case more advanced modelling is unnecessary.

A useful formula for estimating worst case mean concentrations downwind of a point source is the following equation suggested by Hanna *et al.* (1996):

$$C_{wc} = \frac{10^9 Q}{U H_{wc} W_{wc}} \tag{1}$$

where:

- Q = source strength or emission rate of gas or particulate [kg/s]
- C_{wc} = worst case concentration [$\mu g/m^3$]
- U = worst case wind speed at height z = 10 m, usually 1 m/s
- W_{wc} = worst case cloud width [m] (usually assume W = 0.1x, where x is distance from the source)
- H_{wc} = worst case cloud depth (usually assume H = 50 m in worst case)

This equation is essentially a statement of the conservation of pollutant mass, but it illustrates many of the basic parameter dependencies in dispersion modeling. Referring to Figure 2, we assume a uniform concentration in the plume passing through the downwind plane HW. Equation (1) follows from the fact that the flux of pollutant through any plane must equal the source rate Q. Equation (1) illustrates several important dependencies that should be satisfied by all plume models:

1. The mean concentration is inversely proportional to mean wind speed.
2. The mean concentration is directly proportional to the release rate.
3. The mean concentration is inversely proportional to the plume cross-sectional area.

As an example of the above, suppose a small amount (1-kg) of ammonia is released over a period of 30 minutes in an accidental release. Assuming a light wind of 1 m/s, does this release pose any risk to the occupants of a hospital located 5 km downwind? For this example the estimate of the plume width is W_{wc}= 0.1 × 5000 = 500 m, thus,

$$C_{wc} = \frac{10^9 Q}{U H_{\min} W_{\min}} = \frac{(10^9 \mu g/kg)\times(1kg/1800s)}{1m/s \times 50m \times 500m} = 22.2\frac{\mu g}{m^3}$$

This concentration is equivalent to 0.032 ppm, and is 1500 times below the personal exposure limit (PEL) associated with negative health effects due to prolonged exposure to ammonia. Therefore, we can safely say that there is no risk. In such a case there is also no need to perform advanced modelling to assess the risk.

4. The Diffusion Equation and the Gaussian Plume Model

By performing a mass balance on a small control volume, a simplified diffusion equation, which describes a continuous cloud of material dispersing in a turbulent flow, can be written as:

$$\frac{dC}{dt} + U\frac{dC}{dx} = \frac{d}{dy}\left(K_y \frac{dC}{dy}\right) + \frac{d}{dz}\left(K_z \frac{dC}{dz}\right) + S \qquad (2)$$

where: x = along-wind coordinate measured in wind direction from the source
y = cross-wind coordinate direction
z = vertical coordinate measured from the ground
C(x,y,z) = mean concentration of diffusing substance at a point (x,y,z) [kg/m^3]
K_y, K_z = eddy diffusivities in the direction of the y- and z- axes [m^2/s]
U = mean wind velocity along the x-axis [m/s]
S = source/sink term [kg/m^3-s]

Analytical solutions to this equation for the case of dispersion of passive pollutants in a turbulent flow were first obtained in the 1920's by Roberts (1923) and Richardson (1926). The eddy diffusivities (K_y and K_z) are a way of relating the turbulent fluxes of material to the mean gradients of concentration:

$$\overline{v'c'} = -K_y \frac{\partial C}{\partial y}, \quad \overline{w'c'} = -K_z \frac{\partial C}{\partial z}. \qquad (3)$$

Here primed coordinates refer to the turbulent fluctuations of terms about their mean values; for eample, $c(t) = C + c', u(t) = U + u',$ *etc.* Typically in the atmosphere $K_y > K_z$, which explains why the cross-section of a plume often takes on an elliptic shape.

A term-by-term interpretation of Equation (2) is as follows:

$\frac{dC}{dt} + U\frac{dC}{dx}$ Time rate of change and advection of the cloud by the mean wind.

$\frac{d}{dy}\left(K_y \frac{dC}{dy}\right), etc.$ Turbulent diffusion of material relative to the center of the pollutant cloud.

(The cloud will expand over time due to these terms.)

S Source term which represents the net production (or destruction) of pollutant due to sources (or removal mechanisms).

Equation (2) is grossly simplified, since several assumptions are made in its derivation:

1. The pollutant concentrations do not affect the flow field (passive dispersion).
2. Molecular diffusion and longitudinal (along-wind) diffusion are negligible.
3. The flow is incompressible.
4. The wind velocities and concentrations can be decomposed into a mean and fluctuating component with the average value of the fluctuating (stochastic) component equal to zero. Mean values are based on time averages of 10-60 minutes.
5. The turbulent fluxes are linearly related to the gradients of the mean concentrations as in Equation (3).
6. The mean lateral and vertical wind velocities V and W are zero, so we have also restricted our analysis to steady wind flow over an idealized flat terrain.

The Gaussian plume model, which is at the core of almost all regulatory dispersion models, is obtained from the analytical solution to Equation (2). For a continuous point source released at the origin in a uniform (homogeneous) turbulent flow the solution to Equation (2) is:

$$C(x,y,z)=\frac{Q}{4\pi x\sqrt{K_y K_z}}\exp\left(\frac{-y^2}{4K_y(x/U)}\right)\exp\left(\frac{-z^2}{4K_z(x/U)}\right). \quad (4)$$

Unfortunately, the turbulent diffusivities K_y and K_z are unknown in most flows, and in the atmospheric boundary layer K_z is not constant, but increases with height above the ground. In addition, K_y and K_z increase with distance from the source, because the diffusion is affected by different scales of turbulence in the atmosphere as the plume grows (Figure 3). Despite these limitations, the general Gaussian shape of Equation (4) is often. If we define the following Gaussian parameters:

$$\sigma_y=\sqrt{2K_y\frac{x}{U}} \quad \textit{and} \quad \sigma_z=\sqrt{2K_z\frac{x}{U}}. \quad (5)$$

then the final form of the Guassian plume equation, for an elevated plume released at $z = H_p$ is:

$$C(x,y,z)=\frac{Q}{2\pi U_p\sigma_y\sigma_z}\exp\left(-\frac{y^2}{2\sigma_y^2}\right)\left[\exp\left(-\frac{(z-H_p)^2}{2\sigma_z^2}\right)+\exp\left(-\frac{(z+H_p)^2}{2\sigma_z^2}\right)\right] \quad (6)$$

In this expression a second z-exponential term has been added to account for the fact that pollutant cannot diffuse downward through the ground at z = 0. This "image" term can be visualized as an equivalent source located at $z = -H_p$ below the ground, and is further discussed in Section 6 below.

Equation (6) is the well-known Gaussian plume equation for a continuous point source (Turner, 1994). The plume height H_p is the sum of the actual stack height H_s plus any plume rise ΔH due to initial buoyancy and momentum of the release (Figure 4). The wind speed U_p is taken to be the mean wind speed at the height of the stack. Since we are normally interested in concentrations at the ground (where the receptors such as people are), we set z = 0 to obtain,

$$C(x,y,z=0)=\frac{Q}{\pi U_p\sigma_y\sigma_z}\exp\left(-\frac{y^2}{2\sigma_y^2}\right)\exp\left(-\frac{H_p{}^2}{2\sigma_z^2}\right) \quad (7)$$

If we furthermore set H_p= 0 we get the vertical distribution due to a ground-level source:

$$C(x,y,z)=\frac{Q}{\pi U\sigma_y\sigma_z}\exp\left(-\frac{y^2}{2\sigma_y^2}\right)\exp\left(-\frac{z^2}{2\sigma_z^2}\right), \quad (8)$$

It turns out this latter model is not very accurate. For a ground-level release, the vertical profile varies more like $\exp(-z^{1.5})$, rather than the Gaussian form $\exp(-z^2)$, due to the large vertical variations of the diffusivity K_z near the ground (van Ulden, 1978). A more general, non-Gaussian model, which allows for the vertical variation of K_z and the vertical variation of the velocity profile can be written as,

$$C(x,y,z)=\frac{Q}{\sqrt{2\pi}U_p\sigma_y}\exp\left(-\frac{y^2}{2\sigma_y^2}\right)f(z), \quad (9)$$

Here *f(z)* is a normalized function which describes the vertical distribution of material in the plume. To ensure conservation of mass, such models must satisfy:

$$\iint_{y,z} CUdydz = Q, \quad (10)$$

where the integration is taken over the y-z plane, perpendicular to the plume axis.

5. Determination of the Gaussian Plume Parameters σ_y and σ_z

In order to evaluate equations (6) and (7), we require the Gaussian plume parameters σ_y and σ_z. These have been measured as a function of distance from the source in numerous field studies. The most common tabulated data are the Pasquill-Gifford sigmas, which are based primarily on the Project Prairie Grass study in the U.S. (Barad, 1958). The P-G sigmas were formulated by Pasquill (1961) and Gifford (1961) for low-level releases over relatively smooth terrain at distances of a few thousand meters from a source.

The plume parameters σ_y and σ_z are driven by atmospheric turbulence and are influenced by the state of convection in the atmosphere. Atmospheric turbulence is greatly enhanced by convective motions due to heating of the earth's surface. On clear nights, it is suppressed by cooling of the ground due to outgoing radiation. In order to relate the state of atmospheric convection to simply observable parameters, Pasquill developed a simple quantitative rating scheme consisting of six stability classes ranging from highly convective [A] to highly stable flow conditions [F]. These classes are summarized in Table I. The resulting Pasquill-Gifford (P-G) σ_y and σ_z curves under varying conditions of stability are shown in Figure 5.

Table I. Relationship Between Wind Speed, Pasquill-Gifford Stability Class and Monin-Obukhov Length [Hanna, Drivas and Chang, 1996]

Description	P-G Stability Class	Time of Day/Condition	Wind Speed U	M-O Length L_{MO}
Very Unstable	A	Sunny Day	< 3 m/s	-10 m
Unstable	B or C	↓	2-6 m/s	-50 m
Neutral	D	Cloudy or Windy	> 3-4 m/s	|L| > 100 m
Stable	E	↓	2-4 m/s	+ 50 m
Very Stable	F	Clear Night	< 3 m/s	+10 m

For use in regulatory Gaussian plume models, analytic expressions have been fitted to the standard P-G sigma curves. In Appendix A gives the algebraic equations used to calculate σ_y and σ_z in the ISC3 model are provided. These sigma data can be applied for releases over flat, rural terrain. However, dispersion in the urban environment usually produces greater rates of spread than these field data expressions. For urban dispersion, a second set of curves sometimes called the McElroy-Pooler (1968) sigmas, based on tracer releases in a large U.S. city, are used. These are incorporated in the SCREEN3 and ISC3 models when the user selects the urban terrain option in running the software.

The grouping of atmospheric stability into six discrete classes is done in most simple regulatory dispersion models. However, if more detailed information is available, such as directly measured wind velocity fluctuations, then it is possible to relate the plume sigmas directly to these turbulent fluctuations using the statistical theory of diffusion. For example, Draxler (1976) provides the following relationships:

$$\sigma_y = \frac{\sigma_v}{U} x \cdot f_y = \sigma_\theta x \cdot f_y \qquad (11a)$$

$$\sigma_z = \frac{\sigma_w}{U} x \cdot f_z = \sigma_\phi x \cdot f_z , \qquad (11b)$$

where σ_v and σ_w are the root-mean-square fluctuations in transverse and vertical velocities (v and w), and σ_θ and σ_φ are the standard deviations of the wind vector azimuth and elevation angles (in radians). The distance *x* is measured from the source. The functions f_y and f_z are unity close to the source but are decreasing functions of *x*. Thus, if actual measurements of the wind velocity fluctuations are available, σ_y and σ_z can be calculated directly from (11a) and (11b). An approach similar to this is used in the AERMOD dispersion model to calculate the dispersion parameters from atmospheric turbulence (Cimorelli et al., 1998).

6. Plume Reflection at the Ground and at Elevated Inversions

When a plume is discharged from an elevated stack, it will spread vertically until its lower edge reaches the ground. Until this happens, the term: $\exp(-(z+H_p)^2/2\sigma_z^2)$ in equation (6), does not make a significant contribution to concentrations above the ground. However, as σ_z increases, the plume will eventually be reflected at the ground. The virtual source at $z = -H_p$ will then begin to contribute to the aboveground concentrations.

Similarly, if a strong inversion layer is located at some height z_i above the stack, then the plume will have difficulty expanding vertically and will effectively be "trapped" between the inversion and the ground. Plume reflection will occur in this case at both the ground and the inversion layer. To account for the initial reflection at the inversion, an image source is placed at $z = 2z_i - Hp$ as illustrated in Figure 6. This image source will also be reflected at the ground (to ensure $\partial C/\partial z|_{z=0} = 0$), so a further image source is placed at $z = 2z_i - H_p$ below the ground. Similarly, the plume image source at $z = H_p$ below the ground requires an image source above the inversion at $z = 2z_i + H_p$ to ensure $\partial C/\partial z|_{z=zi} = 0$. This process of reflecting image sources can go on indefinitely, but only the first few terms are usually required in the vertical distribution function. The vertical distribution will be non-Gaussian, and we can write:

$$C(x,y,z) = \frac{Q}{2\pi U_p \sigma_y \sigma_z} \exp\left(-\frac{y^2}{2\sigma_y^2}\right) f(z,H_p,z_i,\sigma_z), \tag{12}$$

where:

$$\begin{aligned} f(z,H_p,z_i,\sigma_z) = & \left[\exp\left(-\frac{(z-H_p)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H_p)^2}{2\sigma_z^2}\right)\right] \\ & + \left[\exp\left(-\frac{(z-2z_i+H_p)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+2z_i-H_p)^2}{2\sigma_z^2}\right)\right] \\ & + \left[\exp\left(-\frac{(z-2z_i-H_p)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+2z_i+H_p)^2}{2\sigma_z^2}\right)\right] \end{aligned} \tag{13}$$

In practice, the vertical concentration profile eventually becomes uniformly distributed throughout the mixing layer after which the concentration can be approximated by (Turner, 1994):

$$C(x,y,z) = \frac{Q}{\sqrt{2\pi}U_p\sigma_y z_i}\exp\left(-\frac{y^2}{2\sigma_y^2}\right) \quad (13)$$

This gives reasonable results when $\sigma_z \geq z_i$. This formula also describes the distribution of material during a fumigation episode, which is the brief period when an elevated plume initially above an inversion is mixed downward by convective turbulence as the depth of the mixed layer reaches the height of the plume due to heating at the ground (Turner, 1994).

A buoyant plume is never perfectly reflected by an inversion layer and partial plume trapping is allowed in more advanced models such as AERMOD, where only a fraction f_p of the plume (where $f_p \leq 1$) remains in the convective boundary layer (CBL). The remaining fraction $(1-f_p)$ is allowed to penetrate the inversion and to escape temporarily, but eventually reappears in the CBL if convective conditions raise the inversion cap and diffuse the mass that has escaped back to the ground. The total concentration C_c at a receptor is then given by:

$$C_c = C_d + C_r + C_p \quad (14)$$

where:

C_d = direct source concentration contribution (due to downward dispersion of material from the stack)
C_r = indirect source concentration contribution (due to primary image source above $z_{i,}$. AERMOD includes a delay to mimic the lofting behaviour of a buoyant plume)
C_p = penetrated source concentration distribution, for material that initially penetrates the elevated version

Each of these terms has associated image sources to satisfy the symmetry boundary conditions ($\partial C/\partial z = 0$) at $z = 0$ and $z = z_i$. Further details on this approach are found in the AERMOD model formulation document by Cimorelli *et al*. (1998). During stable conditions, the plume is modelled using only the direct source contribution and reflection at the ground, without "trapping". AERMOD is one of the few models that allows for non-Gaussian distributions of plume material in the vertical.

6. The Plume Advection Velocity and the Wind Speed Velocity Profile

In the lowest part of the earth's boundary layer (the surface layer), wind speed increases with increasing height and has strong gradients near the ground. In homogeneous terrain, under conditions of neutral atmospheric stability, the wind speed is found to vary logarithmically with height (Panofsky and Dutton, 1984):

$$U(z) = \frac{u_*}{\kappa}\ln\left(\frac{z}{z_0}\right) \quad (15)$$

The friction velocity u_* is related to the frictional resistance that the ground exerts on the wind. It is typically about 10% of the wind speed at z = 10 m. The surface roughness length z_0 is a measure of the aerodynamic roughness of the ground, and is typically 3-10% of the height of the surface obstacles (trees, houses, crops, *etc.*). The von Karman constant κ is about 0.4.

The logarithmic wind profile is not easy to work with for dispersion calculations. It is often approximated by a more simple power law of the form,

$$U(z) = U_{10}\left(\frac{z}{10}\right)^p, \qquad (16)$$

The power law coefficient p increases with increasing surface roughness. For different types of terrain, Table II gives the approximate roughness height z_0 and profile exponent p. Some typical wind profiles are shown schematically in Figure 7. Equation (15) is valid only for neutral stability conditions. The exponent p increases dramatically with increasing atmospheric stability Irwin, 1979). The effect of stability on the power law exponent is shown in Appendix B.

Table II. Surface Roughness and Power Exponent for Wind Flow Over Various Terrains in Neutral Stability

Terrain	**z_0 [m]**	**p**
Lake or smooth sea	10^{-4}	0.07
Sandy desert	10^{-3}	0.10
Short grass	0.005	0.13
Open grassland	0.02	0.15
Root crops	0.1	0.2
Agricultural areas	0.2-0.3	0.24-0.26
Parkland/Residential Areas	0.5	0.3
Large Forest/Cities	1.0	0.39

Since windspeed varies with height, it is not obvious what advection speed one should assume when using the Gaussian plume model. In practice, for an elevated source one usually takes U_p to be the mean wind speed at the stack height. Equations (15) or (16) can be used to calculate this velocity from the standard wind speed U_{10} at 10m height. For a ground level source, one typically uses the wind speed U_{10} for the plume advection velocity, although a more precise plume advection velocity for a ground source is given by (van Ulden, 1978):

$$U_p = \frac{u_*}{\kappa}\ln\left(\frac{0.6\sigma_z}{z_0}\right) \qquad (17)$$

This is just the log-law wind speed evaluated at a height z = $0.6\sigma_z$. Because of the dependence on σ_z, it can be seen that as the plume grows vertically, it moves at progressively higher speeds.

6.1 Effect of Atmospheric Stability on the Mean Wind Profile

Along with increased turbulence, one of the effects of atmospheric convection is to modify the shape of the mean velocity profile. A recommended velocity profile function for use in stable and near-neutral conditions is given by (Hanna *et al.*, 1996)

$$U(z) = \frac{u_*}{0.4}\left[\ln\left(\frac{z}{z_0}\right) + 4.5\frac{z}{L_{MO}}\right]. \quad (18)$$

Here L_{MO} is the Monin-Obukhov length scale, defined as:

$$L_{MO} = \frac{-\rho c_p T_a u_*^3}{0.4 g H_f} = \frac{u_*^3}{0.4g\frac{-\overline{w'T'}}{T_a}} \quad (19)$$

where: $H_f = \rho c_p \overline{w'T'}$ = vertical heat flux from ground [W/m^2]
T_a = absolute air temperature [^{0}K]
g = gravitational constant [9.8 m/s^2]
c_p = heat capacity of air [287 J/kg-^{0}K]

Typical values of L_{MO} in various stability conditions are listed in Table I. In a stable atmosphere, $z = L_{MO}$ defines the altitude above which the mechanical production of turbulence is suppressed through the action of negative buoyancy. Thus, when $z < L_{MO}$, the mechanical generation of turbulence is dominant. In a convective atmosphere ($L_{MO} < 0$), $|z/L_{MO}|$ is a measure of the ratio of turbulence production by convection to mechanical production of turbulence. The height $|L_{MO}|$ then defines the boundary between flow levels where mechanical turbulence due to friction dominates ($z < |L_{MO}|$) and levels where convective turbulence dominates the flow ($z > |L_{MO}|$).

Equation (18) can also be approximated by a power law, with the recognition that in this case the exponent p will be a function of z_0 and L_{MO}. Typical exponents are given in Appendix B.

The type of wind profile given in equation (18) is used in the advanced AERMOD dispersion model. AERMOD has a meteorological preprocessor called AERMET which calculates the values of u_* and L_{MO} for a given flow. These are then used in calculating mixing height, temperature and velocity profiles in a convective (CBL) or stable (SBL) atmospheric boundary layer. These factors are then used in the determination of the Gaussian plume parameters σ_y and σ_z. This is a much more sophisticated approach then the traditional methods using charts which only allow for discrete stability classes and roughness categories. The use of Monin-Obukhov scaling of meteorological variables is one of the criteria which differentiates these “advanced” dispersion models from their predecessors.

7. Stack-Tip Downwash and Plume Rise

Most large-scale releases of pollutant are ejected from stacks with initial exit velocity w_s and enough initial buoyancy (due to excess temperature of the effluent) to cause the plume to rise significantly before it is bent over by the wind. On a windy day, the initially vertical plume is quickly bent over by the wind as shown in Figure 9. Because of the relative velocity of the plume and the wind, the plume boundary is very turbulent. The rising plume entrains ambient air and cools before it eventually reaches a maximum height dependant on the ambient environment. Various empirical equations and mathematical models have been proposed for calculating the rise of stack gas plumes in the atmosphere due to this initial momentum and buoyancy.

7.1 Stack-Tip Downwash

If the stack discharge velocity is very small, the stack effluent may be caught in the aerodynamic wake behind the stack and drawn downward (Figure 8). This genrally occurs when the stack exit velocity w_s is less then 1.5 times the mean wind speed U_s at the top of the stack. One way to account for this effect is to introduce an effective stack height H_s' which may is reduced by the amount of downwash according to the following simple equation (Briggs, 1974):

$$H_s' = H_s - 4R_s\left(1.5 - \frac{w_s}{U_s}\right). \tag{20}$$

Here H_s is the physical stack height and R_s is the stack radius. If $w_s > 1.5U_s$, then it is assumed that there is no downwash the term in brackets is set to zero.

7.2 Buoyancy and Momentum Fluxes

In most EPA dispersion models, in order to calculate the plume rise the momentum flux and buoyancy flux parameters are required. These parameters are based on initial exit conditions as follows:

Momentum Flux: $$F_m = w_s^2 R_s^2\left(\frac{\rho_s}{\rho_a}\right) = w_s^2 R_s^2 \frac{T_a}{T_s} \qquad [\mathrm{m^4/s^2}] \tag{21}$$

Buoyancy Flux: $$F_b = g w_s R_s^2 \frac{\rho_s - \rho_a}{\rho_a} = g w_s R_s^2\left(\frac{T_s - T_a}{T_s}\right) \quad [\mathrm{m^4/s^3}] \tag{22}$$

By making several simplifying assumptions about the flow, it is possible to derive simple, closed-form solutions of the equations of motion for the buoyant plume. These include:

1. The wind speed U_s remains constant above the stack.
2. The plume is fully “bent over” (Figure 9) for its entire trajectory.
3. The plume is advected by the mean wind, with $U_p = U_s$.
4. The plume density ρ_p and ambient density ρ_a are equivalent, except in terms involving the difference $\rho_a - \rho_p$ which are used to calculate the buoyancy (i.e., the classical Boussinesq approximation).

The resulting plume rise trajectory in a neutrally stable atmosphere is then given by (Davidson, 1989):

$$\Delta H = \left(\Delta H_m^3 + \Delta H_b^3 + \Delta H_0^3\right)^{1/3} - \Delta H_0 , \qquad (23)$$

In equation (23), ΔH_m is the rise component due to initial momentum,

$$\Delta H_m = \left(\frac{3}{\beta^2}\right)^{1/3} \frac{F_m^{1/3}}{U_p^{2/3}} x^{1/3} , \qquad (24)$$

ΔH_b is the rise component due to initial buoyancy,

$$\Delta H_b = \left(\frac{3}{2\beta^2}\right)^{1/3} \frac{F_b^{1/3}}{U_p} x^{2/3} , \qquad (25)$$

and ΔH_0 is a constant which accounts for the initial size of the plume,

$$\Delta H_0 = \frac{1}{\beta}\left(\frac{T_a}{T_s}\frac{w_s}{U_p}\right)^{1/2} R_s . \qquad (26)$$

The coefficient β is a measure of the rate at which ambient air is entrained by the plume. Both Briggs (1984) and Davidson (1989) recommend using β = 0.6, based on matching the model to observed plume trajectories.

7.3 Final Plume Rise

Equations (23), (24) and (25) suggest that a buoyant plume will rise indefinitely. However, as a hot plume rises, it cools as it entrains ambient air. The plume will eventually reach an elevation where its internal temperature is the same as the ambient air and cannot rise further because the potential temperature θ_a increases with height at the top of the mixing layer. If the plume kept on rising, it would be cooler than the surrounding air and will experience negative buoyancy which pushes it down. This effect can be seen in Figure 9. Thus in a stable atmosphere, the plume rise must be limited. The final rise is determined by the following atmospheric stability parameter

$$S = N^2 = \frac{g}{\theta_a}\frac{\partial \theta_a}{\partial z} . \qquad (27)$$

The characteristic frequency N is called the Brunt-Vaisalla frequency and it is the natural frequency of wave-like motions in the atmosphere. It is found that the maximum rise of a buoyant plume in a stable atmosphere is then given by (Briggs, 1984):

$$\Delta H = 2.66\left(\frac{F_b}{U_p N^2}\right)^{1/3}, \tag{28}$$

Here U_p and N^2 are calculated at the final plume height. If the atmosphere is close to neutral, $N^2 \to 0$ and equation (28) over predicts the final plume rise. In such a case the plume is eventually broken up by atmospheric eddies and a different formulation is necessary, involving the friction velocity u_*. For example, in the AERMOD model the following equation is used:

$$\Delta H = 1.2\left(\frac{F_b}{U_p u_*}\right)^{3/5}\left(H_s + 1.2\frac{F_b}{U_p u_*}\right). \tag{29}$$

Equation (28) is also found to over predict final rise in a stable atmosphere with calm winds. In that case the plume rise is calculated from (Briggs, 1984):

$$\Delta H = 4\left(\frac{F_b}{N^3}\right)^{1/4}. \tag{30}$$

AERMOD takes the final plume rise to be the minimum of equations (28)-(30), which ensures a conservative analysis – *i.e.*, maximum ground level concentrations.

The final rise in an unstable, convective atmosphere with turbulence is generated by solar heating of the ground depends on the following heat flux parameter (Briggs, 1975):

$$H_* = \frac{g\overline{w'T'}}{T_a} = \frac{u_*^3}{\kappa\left|L_{MO}\right|}. \tag{31}$$

In this case the final rise is given by:

$$\Delta H = 4.3\left(\frac{F_b}{U_p}\right)^{3/5} H_*^{-2/5}. \tag{32}$$

This result is based on the idea that the plume rise will terminate when the dissipation rate in the plume has decayed to that of the surrounding turbulent air. This is called the "plume break-up model" (Briggs, 1975).

Prior to reaching the final plume rise, it is correct to use equation (23) in all stability conditions. In calculating the final plume rise using any of the above models, the plume velocity U_p must be evaluated at the final rise height. Since this speed is unknown *a priori*, some sort of iteration is required. Most simple regulatory models simply use the wind speed at stack exit U_s. For a particular plume, there can be considerable error in applying the individual plume rise formulae, since they are based on an aggregate average of many observed plumes. A factor of two error in the final rise height is quite possible, so it is often best to choose the minimum final rise height from a variety of formulas to ensure a conservative estimate of the resulting ground-level concentration.

7.5 Buoyancy-Induced Dispersion

As a plume rises to its final rise height, it entrains air and is spread out by turbulent eddies. During the rise phase, the plume size can be shown to increase linearly with rise height,

$$R_p = R_0 + \beta \Delta H \,. \tag{33}$$

Here R_0 is an effective source radius, which is related to the physical source diameter by (Davidson, 1989),

$$R_0 = \left(\frac{T_a}{T_s} \frac{w_s}{U_s} \right)^{1/2} R_s \,, \tag{34}$$

and β is the usual air entrainment parameter (β ≈ 0.6). The initial rate of plume spreading is greater than that due to ambient turbulence alone. In order to account for the enhancement of the plume spread due to the buoyant rise phase, for example, the SCREEN3 model calculates effective plume spread parameters downwind of the stack according to:

$$\sigma_{ye} = \left(\sigma_y^2 + \left(\frac{\Delta H}{3.5} \right)^2 \right)^{1/2} \tag{35a}$$

$$\sigma_{ze} = \left(\sigma_z^2 + \left(\frac{\Delta H}{3.5} \right)^2 \right)^{1/2} \,. \tag{35b}$$

Here ΔH is the plume rise, and σ_y and σ_z are the P-G sigmas calculated for a point source at the stack (x = 0). A similar approach is used in ISC3 and AERMOD.

8. Initial Source Size and Virtual Source Methods

The tabulated P-G plume sigmas given in Appendix A are based on an ideal point source of pollutant. In practice, we are often interested in area or volume sources of pollution with finite initial lateral and vertical dimensions. In addition, nearby buildings can enhance the initial growth of plumes and can produce an effective volume source in their wakes. As shown in Figure 10, it is possible to account for any initial source dimensions by projecting a hypothetical point source some distance upstream of the actual area/volume source. This is done by first calculating the effective initial plume width σ_{y0} and plume height σ_{z0} due to the area or volume source. The upstream virtual source distances x_{y0} and x_{z0} are then calculated using the P-G sigma formulas. For example, suppose the P-G equation for calculating σ_z for a point source is given by

$$\sigma_z = ax^b, \quad (36)$$

then the virtual source distance x_{z0} is given by:

$$x_{z0} = \left(\frac{\sigma_{z0}}{a} \right)^{1/b}. \quad (37)$$

At later stages of plume development downstream of the source we would calculate σ_z from

$$\sigma_z = a(x + x_{z0})^b. \quad (38)$$

This is equivalent to assuming that the plume originates as a point source a distance ($x + x_{z0}$) upstream. Table III gives recommendation for initial spread parameters based on the dimensions of the actual volume sources. These methods are used in the ISC3 dispersion model.

Table III. Procedures for Estimating Initial Lateral Dimensions and Initial Vertical Dimensions for Volume and Line Sources (ISC3 Model)

Source Type	**σ_{y0} Calculation**	**σ_{z0} Calculation**
Surface-based volume source	Length of side ÷ 4.3	Vertical dimension ÷ 2.15
Elevated source	Length of side ÷ 4.3	Vertical dimension ÷ 4.3
Line source represented by series of volume sources	Length of side ÷ 2.15	Vertical dimension ÷ 2.15 if ground level line source, Vertical dimension ÷ 4.3 if elevated line source

It can be shown that the plume sigmas in Table III are equivalent to choosing the Gaussian plume so that it will have a concentration of 10% of its peak value at the physical edges of the source. This idea is shown schematically in Figure 10.

Although ISC3 uses a virtual source method, AERMOD differs in its treatment of the initial source size. In AERMOD, the initial variance of plume as given in Table III is added to the predicted plume variance for a point source without virtual displacements:

$$\sigma_y^2 = \sigma_{y0}^2 + \sigma_{y,ps}^2, \quad (39)$$

In this equation: σ_{y0} = the initial plume size

$\sigma_{y,ps}$ = the plume size assuming an initial point source

σ_y = the resultant plume size (including effect of initial source size).

8.1 Area Sources and Urban Pollution Problems

Often one is interested in calculating the cumulative effect of numerous small sources (small industries, residential heating, vehicles, *etc*.) that are distributed over a large area. In such cases, the source rate is best expressed as an average pollutant flux per unit area [kg/m^2-s]. Consider a rectangular area source that has crosswind dimension L_y and along-wind dimension L_z. The concentration downwind can be calculated by applying equation (8) to each infinitesimal area source dy×dz and then integrating over the whole area:

$$\begin{aligned} C(x,0,z) &= \frac{q}{\pi U}\int_0^{L_z}\frac{1}{\sigma_y\sigma_z}\exp\left(-\frac{z^2}{2\sigma_z^2}\right)\left[\int_{-L_y/2}^{L_y/2}\exp\left(-\frac{y^2}{2\sigma_y^2}\right)dy\right]dx \\ &= \sqrt{\frac{2}{\pi}}\frac{q}{U}\int_0^{L_z}\frac{1}{\sigma_z}\exp\left(-\frac{z^2}{2\sigma_z^2}\right)erf\left(\frac{L_y}{2\sqrt{2}\sigma_y}\right)dx \end{aligned}, \tag{40}$$

The resulting integral can be expressed in terms of the error function *erf(x)* [1]. For most problems, the area integration must be performed numerically. However in a large city where L_y is large compared to σ_y, the error function is approximately one. In addition, if we are interested only in ground level concentrations (z = 0), then the concentration is given by the simple expression:

$$C(x,0,0) = \sqrt{\frac{2}{\pi}}\int_0^{L_z}\sigma_z^{-1}dx \tag{41}$$

In addition, if the receptor is within the boundaries of the area source of interest, we can replace L_z by the distance *x* from the upstream edge. For the simple case where $\sigma_z = ax^b$, we get the result given by Gifford and Hanna (1970) for concentrations due to urban area sources:

$$C = \sqrt{\frac{2}{\pi}}\frac{x^{1-b}}{a(1-b)}. \tag{42}$$

Assuming b ≈ 0.75 for typical urban dispersion, the dependence of the concentration on *x* (the size of the city) is weak, because the lower layers of air over a city tend to be well mixed. Of course, if there is a vertical limit to mixing due to an elevated inversion, then there will be a buildup of concentration in the atmosphere, and a simple box model calculation yields (for $\sigma_z > z_i$):

[1] The error function erf(x) occurs frequently in probability theory and diffusion problems, and is defined as: $erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$. It is a measure of the area under the Gaussian distribution function.

$$C \approx \frac{q}{U_p} \frac{x}{z_i}. \tag{43}$$

This last result explains why air quality in the center of large cities is generally very poor on calm days with a low-level capping inversion that traps the pollutants near the ground.

9. Complex Terrain Algorithms

Although the derivation of the Gaussian Plume model assumes ideal conditions such as an infinite, flat, homogeneous area, the Gaussian plume model is often used to predict concentrations at receptors in complex, elevated terrain. There are several ways to account for these effects, and the method applied depends somewhat on the atmospheric stability class and whether the "elevated simple terrain" or "complex terrain" option is selected in the model. SCREEN3 and ISC3 are both similar in their approach to terrain modelling. The approach used in AERMOD is more sophisticated. In all these models, only the vertical distribution function is affected. With the exception of the CALPUFF model, there is no mechanism for lateral deflection of the plume due to terrain in these standard regulatory air dispersion models.

9.1 Elevated Simple Terrain

In SCREEN3, the elevated simple terrain option is chosen when the terrain height at the receptor is above the stack base elevation, but below the release height. The net plume height above sea level, which includes the effect of plume rise (i.e., $H_p = H_s + \Delta H$), is then kept constant, i.e. it does not follow the terrain contours. Instead, the plume height above ground at a point receptor (x,y) downwind depends on the local elevation z(x,y). The modified plume height above the receptor is calculated as

$$H_p' = H_p + z_s - z(x,y) \tag{44}$$

where:

- H_p' = the effective plume height above the receptor
- H_p = uncorrected plume height above the base of the stack
- z_s = elevation of stack base
- $z(x,y)$ = elevation of terrain at the receptor location

If the receptor is located above the ground, then the effective plume height above the receptor is reduced by the receptor height above ground (the so-called "flagpole" height), and also by the local terrain elevation above the stack base. This is shown in Figure 11.

9.2 Complex Terrain

When the complex terrain option is selected in ISC3, the plume is assumed to be terrain following. The effective plume stabilization height above the ground at a point (x,y) is then given by:

$$H_p' = H_s + \Delta H - (1 - f_t)H_t, \tag{45}$$

where: H_s = stack height
ΔH = plume rise assuming flat terrain
H_t = terrain height of the receptor location above the base of the stack.(= $z(x,y) - z_s$).

The terrain adjustment factor is stability dependent:

$f_t = 0.5$ for stability categories A-D
$f_t = 0$ for stability categories E, F.

In neutral and unstable conditions (categories A-D), the plume stabilization height is adjusted to partially follow the terrain. However, under stable conditions the plume height is not deflected by the terrain. The plume axis is assumed to remain fixed at the plume stabilization height above mean sea level. If the plume encounters an obstacle such as a hill that is above this height, then the plume can impact on the obstacle, leading to high concentrations. As a result of the terrain adjustment factor ($f_t = 0.5$), during unstable and neutral conditions, the plume height relative to the stack base is deflected upwards by an amount equal to half the terrain height as it passes over complex terrain, Figure 12. This concept is based on a recommendation by Egan (1975), who found from the results of wind tunnel experiments and potential flow theory that the wind streamlines never fully follow the underlying terrain.

9.3 AERMOD Approach to Complex Terrain

In the AERMOD approach to complex terrain, the terrain features are first processed by AERMAP (the terrain preprocessor), which assigns an effective height of terrain h_c to each receptor point (x_r, y_r, z_r). In order to calculate the concentrations at downwind receptors located on elevated terrain, AERMOD performs two concentration calculations corresponding to two extreme plume states. These are: 1) a horizontal plume as under very stable conditions such that the flow remains horizontal (and may impact on a large hill); and 2) a plume that completely follows vertical terrain (terrain following state), so that its centerline height above the local terrain is equivalent to the plume height above flat terrain. The concentration at a receptor C(x_r, y_r, z_r), is then calculated as a weighted sum of these two contributions,

$$C_T = f_t \times C_{h,s} + (1 - f_t) \times C_{t,s}, \tag{46}$$

where: C_T = total weighted concentration
$C_{h,s}$ = concentration due to the horizontal plume state
$C_{t,s}$ = concentration due to the terrain following plume state (*i.e.*, the plume rises with the terrain)
f_t = terrain weighting factor.

The weighting factor in this case is a function of the fraction of plume mass that lies below the dividing streamline height H_c[2]. In this case, the parameter f_t is in the range 0.5 – 1, which means that the plume never completely approaches the terrain-following state. There is always some contribution due to the horizontal plume state. For further details on this approach, the reader is referred to the model formulation document by Cimorelli *et al*. (1998).

10. The Effect of Buildings and Building Wakes on Plume Dispersion

Many industrial stacks are located on top of buildings or in plant sites where there are large buildings nearby. One of the major challenges in regulatory dispersion modeling is to account for the effects of buildings on the near-field dispersion of a plume. There is often a single, large structure that dominates the scene, such as a nuclear reactor building. Most research on this problem had been done for releases on or nearby individual buildings.

The dispersion patterns around isolated buildings are generated by several flow features and are a function of obstacle shape, approach flow turbulence and wind direction. The time-averaged flow field around a rectangular building in a turbulent shear flow is shown in Figure 13, taken from Hosker (1979). Within this flow, which is very unsteady, there are five fundamental regions to consider:

1. A displacement zone, where the incident wind is first influenced by the building
2. A region of separated flow over the upstream edges (roof cavity zone)
3. A ground-based "horseshoe" vortex system that wraps around the base of the building
4. A wake cavity of recirculating flow behind the building.
5. A slowly decaying wake with reduced mean velocity and enhanced turbulence.

The flow field in figure 13 results when the wind blows normal to a face. When the wind approaches the building on a diagonal, a strong elevated trailing vortex may be generated.

As a rule of thumb, if a nearby source is released from a height greater than two-and-one-half times the building height, then there is usually no significant influence of the building on the dispersion (Hanna et al., 1982). For sources nearer the building, the regions of flow separation, trailing vorticity and enhanced turbulence as shown in Figure 14 will likely affect the plume.

<u>10.1 Separated Flow</u>

Separation occurs at the sharp upwind edges of bluff obstacles embedded in a turbulent flow. The regions of separated flow are characterized by low wind speed, high turbulence, high velocity gradients, and flow reversal. To quantify the size of the separated flow regions around a building, a representative building length scale can be defined as (Wilson, 1979):

$$L_B = B_S^{2/3} B_L^{1/3}, \qquad (47)$$

[2] The dividing streamline H_c is the theoretical streamline height which divides the flow into two layers, one which remains horizontal and one which rises over the terrain feature. This concept comes from experiments involving flow over hills in stratified flow.

where: B_S = smaller of upwind building face dimensions H or W
B_L = larger of upwind building face dimensions H or W.

Depending on the length of the building, the roof top cavity may or may not reattach before the rear edge of the building. The length and height of the rooftop recirculation cavity is estimated as (Wilson, 1979):

$$L_c = 0.9 L_B, \quad H_c = 0.22 L_B . \tag{48}$$

Therefore the roof cavity will reattach to the roof as in Figure 14 if $L_c < L$, where L is the along-wind length of the building.

The exact dimensions of the recirculation region downwind of an obstacle depend somewhat on the intensity and scales of turbulence in the approaching flow. However, for a broad range of building dimensions, the mean length of the "wake bubble" can be calculated using the following formula (Fackrell, 1984):

$$\frac{L_R}{H} = \left(\frac{L}{H}\right)^{-0.3} \frac{1.8(W/H)}{1+0.24(W/H)}, \qquad 0.3 \le \frac{L}{H} \le 3.0 \tag{49}$$

For a building in the shape of a cube, $L_R/H \approx 1.5$. Within the wake bubble, the low speed flow recirculates and the instantaneous wind velocities fluctuate randomly so that any pollutants are rapidly dispersed throughout the cavity region. There is a characteristic delay time before any entrained pollutant is released to the outer flow (the "residence time").

Downwind of the wake cavity the wind begins to return to the conditions of the approach flow, but with reduced mean speed and enhanced turbulence. This generally leads to a greater rate of dispersion for plumes that are entrained in the wake. The crosswind velocity defect, which is approximately bell-shaped, extends to 10-20H downwind, depending on the W/H ratio of the building. It is shorter for narrow buildings, longer for wide buildings (Peterka *et al.*, 1985).

10.2 Frontal Eddy and Trailing Vorticity

When a bluff body is placed in a shear flow, there is a tendency for a frontal eddy to form at the upwind face. Because the stagnation pressure increases with height above the ground, there is a downward flow induced on the lower 2/3 of the upwind obstacle face as seen in Figure 14. The frontal eddy trails around the sides of the obstacle and creates a trailing vortex with longitudinally oriented vorticity (Figure 13). This vortex can induce a "downwash" flow that draws the plume from rooftop stacks downward.

At acute angles to the wind, the rooftop leading edges of the building can produce longitudinally oriented vortices that transport the higher velocity air from above down into the central portions of the wake. This effect can extend some 50-100H downwind of the building (Hosker, 1984), and can contribute to downwash of plume material into the wake, and increased lateral spreading.

10.3 Effect of Building Downwash and Wake Flow on Plume Dispersion

There are several available models to account for the enhanced dispersion of effluent caused by buildings located near a stack. Most of these are empirical correlations based on the results of controlled wind tunnel experiments and more limited field experiments. In many simple models for rooftop stacks, the plume rise due to momentum is calculated at a distance of two building heights downwind, ignoring the effect of the building. If this plume height is less than some criteria (*e.g.*, some multiple of the building height) then the plume is assumed to enter the wake cavity. In cases where the source is right on or very near the building, the modified plume rise algorithm due to Schulman and Scire (1980) can be used to calculate the initial rise.

If a plume is entrained in a building wake, the SCREEN3 model calculates a cavity concentration using the following formula (Hosker, 1984),

$$C_c = \frac{Q}{K_c A_f U}, \qquad (50)$$

where: Q = emission rate
A_f = building cross-sectional area normal to wind (H×W)
U = reference wind speed (typically at 10 m height)
K_c = non-dimensional concentration constant (= 1.5).

Since the size of the wake cavity depends on the building orientation relative to the wind, the cavity dimensions should be calculated for at least two extreme building orientations in order to get a reasonable bound on the cavity concentration estimate.

When a plume is fully entrained into a building wake, it starts with an initial size approximately equal to the building cross-sectional area A_f. This building-enhanced dispersion can be accounted for by virtual source displacements:

$$\begin{aligned} \sigma'_{y0} &= \sigma_y(x + x_{y0}) \\ \sigma'_{z0} &= \sigma_z(x + x_{z0}) \end{aligned} \qquad (51)$$

The virtual distances x_{y0} and x_{z0} are chosen so that the initial value of the dispersion parameters at the rear face of the building (σ'_{y0} and σ'_{z0}) are some fraction of the building width and height, respectively. A simple model recommended by Turner (1969) is:

$$\sigma'_{y0} = \frac{W}{4.3}, \quad \sigma'_{z0} = \frac{H}{2.15}. \qquad (52)$$

These virtual source estimates are illustrated in Figure 10.

For building-affected dispersion calculations in the wake, ISC3 and AERMOD use the Huber-Snyder (1982) model to calculate the enhanced dispersion for sources entrained in the wake. Neither of these models will predict concentrations directly in the wake cavity (x < 3H), however SCREEN3 can be used for that. The Huber-Snyder model has different expressions for the near wake (3 < x/H < 10) and far wake (10 < x/H) regions. In the near wake, the dispersion parameters depend on the building dimensions, with:

$$\begin{aligned} \sigma_y' &= 0.35W + (x-3H)/15 \\ \sigma_z' &= 0.7H + (x-3H)/15 \end{aligned} \qquad (53)$$

In the far wake, the P-G sigmas are used with virtual source displacements x_{y0} and x_{z0} chosen so as to match the plume dimensions at x = 10H. This is done in SCREEN3 and ISC3 models. In AERMOD, which uses a more complicated method of calculating the plume sigmas, the plume variances are added:

$$\sigma_{yT}^2 = \sigma_y^2 + \sigma_y'^2 \quad and \quad \sigma_{zT}^2 = \sigma_z^2 + \sigma_z'^2, \qquad (54)$$

where:

σ_{yT}^2 = total variance of the plume

σ_y^2 = Pasquill-Gifford variance calculated for a point source

$\sigma_y'^2$ = Huber-Snyder variance evaluated at x = 10H

Special cases may arise when applying equation (53). If W >>H (a squat building) then H replaces the value of W in calculating σ_y'. For a very tall building (H >>W), W replaces H in the σ_z' equation, otherwise unrealistically high vertical dispersion is predicted. Further details on incorporating the H-S model are described in the ISC3 Model documentation.

The Huber-Snyder enhanced dispersion model is only applied if the plume height, evaluated at two building heights downstream, is less than H + 1.5×min(W,H), where min(W,H) is the lesser of the building frontal dimensions. The ISC3 model modifies only σ_z if the plume height is greater than 1.2 building heights. If the building-affected sigmas from Equation (53) are less than the P-G sigmas at the same distance downwind, then the latter are used.

In ISC3 and AERMOD, if the stack height is less than H + 0.5×min(H,W), a more sophisticated algorithm due to Schulman and Scire (1980) is used to calculate the downwash effect because the initial rise is reduced by the building-enhanced dilution. This model is described in some detail in the original reference and in the model formulation documents for ISC3 and AERMOD.

The overall effect of enhanced dispersion in building wakes is to decrease the maximum ground-level concentrations for low level releases or to increase the ground level concentrations for elevated sources. Using extensive wind tunnel and field data, Fackrell (1984) found that the differences between the measured and predicted concentrations using the Huber-Snyder and other simple building-affected dispersion models was often less than a factor of two or three.

This is within the expected limits of accuracy for most Gaussian plume models (Beychok, 1995). However, none of the models seems to cope well with acute wind angles, where the trailing vortices can induce severe downwash and increased ground level concentrations even for more elevated sources ($H_s > 2.5H$). One promising approach is that used by the ADMS model (Carruthers *et al.*, 1994). In this treatment of buildings, a rooftop plume is partitioned so that only a fraction of the plume is entrained in the wake. This part is then accounted for by locating a virtual source upwind (Figure 15). The part of the plume that is not entrained is treated as an elevated point source. The net concentrations downwind are found by adding the two concentration distributions due to the wake and elevated source.

11. Summary

Although the Gaussian plume model (GPM) is based upon many simplifying assumptions about the dispersion process, it is applied to a wide array of dispersion scenarios and some form of this model is adopted in most regulatory air pollution models for continuous releases. In order to extend the applicability of the GPM to realistic scenarios, the U.S. EPA models make use of several special algorithms or semi-empirical corrections to account for the various effects. These include the influence of atmospheric stability, plume trapping below elevated inversions, fumigation, non-uniform wind profiles, dry or wet deposition, stack-tip downwash, buoyancy-induced dispersion, finite initial source dimensions, complex terrain and the influence of buildings. A brief summary of the algorithms used to incorporate these various features has been provided in this review.

As shown in this review, many of the algorithms in the advanced EPA models are based on simplified physical models of the various dispersion processes, combined with empirical data. The modifications to the basic GPM make extensive use of wind tunnel and measured field data. Because of strong peer reviews and model validations studies, the resulting model codes are quite robust and can be used in a wide variety of situations combining many separate effects.

All of the EPA codes, such as SCREEN3, ISC3, AERMOD and CALPUFF, have been run through extensive physical audits, sensitivity analyses, and quality assurance studies using benchmark data in order to justify their use in environmental assessment. An understanding of the fundamental concepts used in these models is important for the most intelligent use to be made of the models. This background knowledge is required to ensure that the most sensible choices are made in all aspects of the data input stages, selection of model options and the interpretation of results. To become an “expert” user, it is essential that one read the model formulation documentation and some of the references in this document.

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